DEVELOPMENTS IN PHILOSOPHY IN/OF MATHEMATICAL EDUCATION: EPISTEMOLOGICAL, ONTOLOGICAL ANTHROPOLOGICAL QUESTIONS POSED BY THE PRESENCE OF COMPUTERS AND OTHER MEDIA IN MATHEMATICAL EDUCATION PRACTICE

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Abstract

This essay discusses the development of research that the authoress has carried out on the theme for approximately ten years. The philosophical thought evolved in the realm of Philosophy of Mathematical Education is presented. Studies on Mathematical Education as realized in the cyberspace and, based on these investigations, examinations are proposed, comprehensions are exposed, and questions are framed about the reality of cyberspace. Epistemological and anthropological aspects present in the dynamic of being-with-computers and other media are addressed from an intertwined standpoint, seen as an entanglement that reveals the complexity of the worldly reality in which we live and where cyberspace makes itself present. It is in this entanglement that Mathematical Education takes place, and that the Philosophy of Mathematical Education aims to conduct analyses, critiques, and reflections.

Key-words: Philosophy of Mathematical Education; cyberspace; ontology; epistemology; anthropology

TODAY, educational computer-based activities that resort to these devices as well as to similar media are all centered on the informational screen. In the past 30 years, we went from working using computer resources as tools with specific ends in mind, to the notion that, when we develop activities based on the computer and related media, we are actually in side-by-side terms with them, establishing a dialectic cognitive relationship that reorganizes thought (Tikhomirov, 1976; Borba, 1999; Borba & Villarreal, 2005).

I understand that it is the role of the Philosophy of Mathematical Education to analyze and ponder about this reality, shedding light on the meanings and senses that emerge in works of Mathematical Education authors, especially those who address the Technologies in Mathematical Education in their research.

I have been trying to understand the ontological and epistemological aspects that, mandatorily, open up into anthropological questions. These are aspects that have to be appropriately captured, when our attention falls on the reality called "virtual" by authors such as Lévy (1996), among others. It came to my attention that, by the year 2000, Lévy and other researchers, in their definition of virtual, had in fact described what was not virtual, as defining it in opposition to the real.

The present text is a summary of an *essay* about the studies on this subject that I have been carrying out for almost ten years (BICUDO, 2014). Yet I must warn that, instead of laboring the definition of *virtual reality*, I will address *cyberspace* and, in order to comply with the clarity required in every Philosophical text, more specifically the Philosophy of Mathematical Education, I will disclose my thoughts about that word in my clarifications and argumentations. I should add that I understand cyberspace as a

worldly reality materialized in the historicity of the life-world[1], as coined in Husserlian philosophy[2], and that will likewise be clarified. In the present *text*, I will mention the following aspects: the ontological aspects of cyberspace; the epistemological aspects in cyberspace; the anthropological aspects that manifest and become stronger in cyberspace. In this text these aspects are intertwined. They should not be seen independently from one another; rather, they are to be considered an entanglement that unveils the complexity of everyday reality in which we live in, and in which cyberspace becomes. It is in this entanglement that Mathematical Education realizes itself and the Philosophy of Mathematical Education endeavors to analyze, to critique, and to reflect.

The ontological aspects of cyberspace
Here, the discussion on the theme depicted in the title of this section is a comprehensive summary of the topic discussed in the book by Bicudo & Rosa (2010). In order to understand the questions that are specific to ontology, we inquired, in that text, into the *where* in which the subjects intentionally attentive to the informational screen meet the computer program, as supported on the same screen. We recognized that, in this cyberspace, people get involved with one another from different perspectives, like the emotional, the cognitive, and the commercial standpoints, forming an intersubjective community. When we inquire into *where* this meeting takes place, our attention lies, deliberately, on the spatial question. The *where* in which subjective and intersubjective experiences occur, either mediated or side by side with media, is considered by virtual reality authors such as Castells (2005), Lévy (2005), Turkle (1995), Likauskas (2005), Lopes (2005) not as the *real*, but as the *virtual*, since this *where* is not shaped in the dimensions of the physical world space, such as conceived in Classical Physics. Lévy (2005) maintains the de-territorialization of space, since here one is with the computer, and the actions that unfold actualize realities in unimagined settings. The authors above perceive the difference, understanding that there is a space where meetings indeed take place that, in spite of that, they name *virtual*, to differentiate it from the *real space*. As for time, Castells (2005) refers to the *atemporality* of time, denoting an ambiguity that is common when one is on the verge of not accepting a concept and yet no other notion has unfolded. So, to clarify my thoughts: if it is time, it cannot be atemporal; otherwise, how can I declare the existence of time, without time? In other words, what is the concept of time that the author may be referring to, when he alludes to the atemporality of time? I gather that the author may have meant to say *a time that is not linear or chronologically measurable.*

What is shown is that the concept of time and of space, such as in the Classical Physics model, does not explain what is seen happening in cyberspace.   Classical Physics deals with the concept of *real* as what is objectively given and exists in its own right, what is possible to be measured in space (in the three dimensions: height, depth, and width), and time. It is a spatial and temporal totality, where all people and things are placed, and where history and social facts happen. The Cartesian space, with two input variables (space and time), affords to locate precisely where the event or the object are.

However, in the cyber world, the *where* does not fit in this Cartesian space. This is due to a variety of reasons. In it, we are unable to point to the locations people or ideas meet, the intersection of two axes — space and time — since this *where* unfolds along fast and dynamic connections that branch out to yet more and more unpredictable

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[1] Life-world is the world, taken in its totality, where we realize our experience, interleaved in spatiality and temporality; it is a reality constructed in the historical and cultural moment that brings together the present, the past, and the future. I will explain this term further below.

[2] I note that the authors cited, such as Husserl and Heidegger, are phenomenologists.

connections. This concept of space-time is gradually experienced in daily life, as far as we are interwoven in complex events whose triggers are not detectable, unless we resort to sophisticated investigative softwares, which in turn have to be handled by specialists. We sense ourselves in the shift of events, we perceive that we prompt actions that are transformed into messages and that certainly leave their own cultural imprint. However, at the same time we wake up to the fact that we are moving across a historical-cultural ground. To my understanding, it is necessary to open up to the concept of space-time as used in Contemporary Physics so as to begin to comprehend the life-world as we experience it today, and to see that cyberspace manifests side by side with the physicality of nature.

In Bicudo & Rosa (2010), we present our notion that Quantum Physics and the Theory of Relativity, in that they put to the test the notions instituted by Modern Science, actually help us understand reality from distinct perspectives. Space and time can no longer be treated separately; they have become part of the action. In turn, action creates reality, such as revealed in the notion of *quiff* — understood as a function of the quantum wave. For us, this is the trigger of the event, and therefore expands space. When we collect these ideas together in cyberspace, we understand that cyberspace reality is better apprehended in terms of the four-dimensional continuum in which space-time are inseparable and where the action carried out by the subject and enabled by the computer creates spatialities and temporalities. Hence our statement that what is considered virtual in cyberspace is, merely, the *real.*

Understanding cyberspace as real

Next I intend to discuss aspects that shed more light on the notion cited in the paragraph above, and that, in cyberspace, the *real* is not *virtual*, that is, cyberspace is not to be characterized as virtual, since it is but a mode of the real. With that in mind, I briefly will refer to the concepts lodged in the History of Western Philosophy, and adopt the concept nurtured by Granger, quoting his work, published in 1995 (Granger, 1995) on the Philosophy of Science.

The interrogation surrounding virtual and real has been addressed in the History of Philosophy for quite a while now. Such is an ontological issue, when the question that emerges is *what is it, then, the real*? The *virtual* transcends the pragmatic aspects inherent to focusing on the real as a mere location, bestowed with geophysical characteristics and palpable and practical concreteness. Aristotle explains the real as a constant movement of potency and act, form and matter. Two are the pairs: *potency and act*, and *form and matter*; however, these are not synonyms or similar in their characteristics; instead, they intermingle in the occurrence of the real. The real oscillates between pure potency, which it is not, since it is not actualized, and pure form, which has nothing of matter (Mora, 1994). That is why the Greek philosopher uses the two pairs, since, in the reality where we live, there is the act that, when triggered, starts the actualization process of potency.

This complexity is also addressed by Granger (1995), who, in the 1990's, studies the Philosophy of Science based on Aristotle's ideas. Back then the author develops the concept of *present*, understood as the actualized, and of *non-present*, which includes the virtual, the possible, and the probable, that is, what may happen, but has not yet. This occurrence may be understood from a number of standpoints. The author discusses several ideas, some of which, as I understand them, are important to comprehend information technology, especially those in the context of Physics and other sciences. He understands Mathematics as virtual, since, through serial abstraction processes, the forms with which Mathematics works are *forms in general*, in an ontology of forms that

are not directly abstracted from the empirical experience. This science covers a wide domain, and encompasses invariants that are not accountable to actualization of *forms in general* and, at some point, Mathematics covers also the forms of empirically actualizable objects. For Granger, the *forms in general* with which Mathematics works are, therefore, virtual, possible, and probable, and may actualize themselves in actions (acts) triggered and intertwined to the materialities and techniques (matter) available, as well as in particular applications that are approximate probabilistic explanations of what is empirically presented, and so on. Reality in Mathematics is virtual in the sense that it does not depend on empirical contents to be shown, though it depends on formal contents. The connection between the virtual aspect of Mathematics and the empiricism of Natural Sciences is effected through the scientific-theoretical system of references that supports modes of applicability. If we take an object in the realm of Mechanical Physics as an example, we see that its reality is determined by the theoretical referents in its coordinates and that, for that reason, its actualization is intertwined in a finite number of elements. In this sense, the *actual* of the product of this Science is determined, albeit incompletely, since it is more than the general form (virtual) of Mathematics, given that it realized the materialization of its product with the actualizing acts (acts) and with the technical-scientific-technological materiality (matter) available, being, at the same time, less than that general form (virtual), because it does not present it completely. The realized product, therefore, carries the virtual, the possible, and the probable. This complexity is called *informational screen*, which sustains the scientific-technological actualization. It is not an inflexible screen, which would determine the invariants of actualization, by lodging the acts and the available materiality. The impossibility of completely realizing the virtual of the general form in Mathematics is transcended by the pluralism and multiplicity of possibilities in Natural Science.

The concept of *non-actual* and of *actual* led us, Rosa and me (Bicudo & Rosa, 2010), to understand the reality of cyberspace. We understand that the reality of cyberspace is a complexity in which the virtual (general form of Mathematics), the possible, and the probable (the scientific-technological apparatus), the act (the actualizations triggered by the actions of the people who work with the informational screen) are present. The actualization is realized by the acts of the people who act according to their own traits, whether they are imaginative, emotive, cognitive, or judgmental, when they operate with the informational screen. In light of the scientific-technological apparatus that backs computers and other media, we see that a networked actualization takes place, branching out smoothly and quickly, connecting people that communicate through a specific language determined by a reference system (computational programs) and their own acts, laden with their own traits. Here, questions transpire as to the ways humans adopt when they are with the informational screen (computers and other media). To me, this is a crucial issue in philosophical thinking, since it requires us to take the pathways of epistemology and of anthropology.

The interweaving of anthropological and epistemological aspects
These aspects concern the ways the human being is and knows. It is important to make clear that I do not refer to an abstract and generalized concept of man, let alone of a theory of knowledge that may support possible explanations. I am framing questions and explanations about the different forms of the understanding of what it is to be with the informational screen, knowing oneself, knowing and producing knowledge. I see that we live in the life-world, the *Lebenswelt*, as put by Husserl (1970), that is, the world, taken in its totality, where we realize our experience, interleaved in spatiality and temporality. Life-world is a reality constructed in the historical and cultural moment

that brings together the present, the past, and the future. It is not a vessel in which we are placed or in which we drop knowledge, theories, etc, as if these were objects in their own empiricism. Rather, these are the spatiality and the temporality in whose dimensions we live with others, whether human or not, whose reality we in turn weave using articulated comprehensions, subjectively and intersubjectively, that are materialized in available forms and contents. What is intersubjectively understood and is kept via the repetition of successful actions forms itself, gradually, through the intertwining of senses and meanings in objectualities[3].   I understand the constitution of life-world as the constitution of the ground, as a network, where we are. So, this is where we find ourselves, the moment we are with the computer or other media. This involves what there is in the spatiality/temporality in the actual and non-actual modes and, therefore, also involves the cyberspace. It is in the wake of this comprehension that I frame questions and discuss the understandings about the way of being and knowing ourselves at and with the computer and other media. In Being and Time (Heidegger, 1962), when exploring the unity being-with-the-other, Heidegger points to an existential totality, in which the Dasein (the human being seen as each one of us, in each case) is always in spatiality, that is, it is in the there, and it always is with the other (that is, anything, a person or otherwise). According to this way of thinking, the *with* is a *determination* of the Dasein, whose possibilities of becoming diversify, in line with what is, and in line with the way it is, since, given that its character of *being-there-in-the-world-with*, it does not require processes that are otherwise used to establish relationships in order to place itself together with, being with, though it is always with. As for the being-with-the-computer, we see that the person does not place him/herself with it, the computer, as in *being-with* in a mode of *preoccupation*, as Heidegger says (1962) but there is a preponderance of the way of *being-with*, as an *occupation*. Nevertheless, it is not possible to use Heidegger's explanation (Heidegger, 1962) about being with tools available. The reason is that, as said above, there is a dialectic that supports an exchange between the person and the computer and that accepts the idea of reorganization of thought as it is understood by the theory of activity, as, for example, Thikmirov says about it, and, to some extent, of dialogue too. Also, proceeding along this train of comprehensions, it is possible to invoke intersubjectivity[4]. In these articulations, the complexity of the reality in which we live becomes clear as the complexity of knowledge processing. The cognitive process is enhanced also by the logic of computer and its programs as the subject works with them. The dialogue establishes itself between subjectivities, enhanced by the computer, also. In it the historicity of each person presentified in the messages sent and that elicit answers by the receptors, which are filled with their recollections, rendering their historicities and the computer and other media alive, when the informational screen that prints shapes also reorganizes thought. I understand that this process also occurs when the dialogue is established with person-computer-avatar, when computational interfaces are called into play, which are important, if not essential, since they interconnect human-computer (Figueiredo, 2014, p. 138).

Mathematical Education realized in the cyberspace
I understand that the complexity of the reality of cyberspace, here discussed in ontological, epistemological, and anthropological terms is being perceived by authors

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[3] Objectualities are objectivities built on the shift of subjectivity-intersubjectivity and, therefore, do not concern objectivity separately from this shift. This will be further clarified in this section.

[4] This subject will be addressed again later.

who study the theme Mathematical Education as realized with the computer and other media. Numerous research efforts have been developed to elicit the mode through which teaching and learning of Mathematics take place in cyberspace. I understand that these efforts contribute, little by little, to elucidate the mode of being in this spatiality and temporality, teaching, learning, and producing Mathematical Education and Mathematics. What becomes evident in the process of change is the way of thinking logically. For me what is more astonishing is the way of living temporality. The children in the early education stage (between 3 and 6 years of age, in Brazil) does not show tolerance, and waiting is not accepted. As Barreto & Nascimento says (2014), time spent waiting is time to be spent doing something else. The flow of time is not lived: present, past, and future are not experienced. Everything is about the *now*, the instantaneous present. Places, scenarios, situations are replaced within a click. Spatiality is also experienced with-the-computer and other media, at the speed of a trip.

Mathematical Education actualized in the cyberspace must put on evidence the logical thinking as well as the way of being in the world with others.

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