Research procedures to understand algebraic structures: a hermeneutic approach

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Abstract: *The authors question how hermetic texts in mathematics, history, and philosophy of mathematics, specifically those referring to abstract algebra, can be open to the understanding of researchers, teachers and students who are intentionally willing to understand them. This discussion suggests that the openness may happen through hermeneutic procedures. These procedures, in the wake of Gadamerian thinking, take the dialectic of the process of formulating the question and pursuing possible ways of answering it, going into the intricacies of the historicity of knowledge constitution based on the Husserlian perspective and the production of algebra itself, in order to explain a methodology of research. They advance by bringing a hermeneutic study on algebraic structures, explaining the research methodology that aims at the formation of the ideality of algebraic structures and their maturation in the living present as notions of algebraic structures, as objects of study and as subjects of algebra.*

Keywords: algebraic structures, phenomenology, hermeneutics

INVESTIGATION PROCEDURES

This research addresses questions regarding algebra, interrogating its structures.

The word “structure” will be mentioned many times throughout this text. However, it is not a matter of simply repeating it to exhaustion. Structure refers to a concept, which when taken generically, concerns the organization, arrangement and order of essential elements that comprise an entity (concrete or abstract). In the present work we refer to the body of knowledge of algebra and objects, *algebraic structures,* presented within mathematics as definitions. For instance, Mac Lane’s (1986, p. 23) definition of *group*, from a mathematical standpoint shows what the *structure group* is. For him:

> A group is a set G equipped with three rules:
>
> (i) A rule assigning to any two elements s, t of G an element st, called their product, such that the product is associative, r(st) = (rs)t, for all r,s t in G.

(ii) A rule assigning an element e and (the unit, often written as e = 1) of G such that, for all t in G, te = t.

(iii) A rule assigning to each t in G an element t^{-1} in G so that t^{-1} = e.

This definition of *group* shows the understanding of the structure of *group* – an algebraic structure – that expresses the intertwining of compositions, analogies and mathematic procedures producing a set of dependencies that are articulated and unveil their own organization.

By approaching the theme of *algebraic* structures as the focus of the present research, we seek to find a direction that allows us to weave a conducing thread of mathematical activities and evidence which are amalgamated in the construction /production of knowledge of *algebraic structures*. The thread followed in the exploration of the theme is constituted by the numerical field. However, in this text, when we refer to the word *structure*, it does not refer solely to *algebraic structures* as understood in the body of knowledge of mathematics, but also to the invariants which persist in the movement of their construction/production, the way they reveal themselves and are organized within the body of knowledge which is algebra. Therefore, one can talk about *structure* in different contexts of development of algebraic theory, as well as, envision a ***structure*** of *structure*s, which shows the way any and all *structure* is constituted: as organization of the invariants highlighted by the articulation of thought, ways to designate them and how to proceed in terms of mathematical operations.

In the movement of pursuing algebraic structures we assumed the vision of phenomenology regarding investigation procedures and knowledge. We worked with the ideas presented by Husserl, Heidegger and Gadamer, specifically intertwining those concerning history, hermeneutics and thought. From Husserl, we emphasize the ideas regarding history, understood as the living movement of original formation of meaning and its sedimentation. Its clarity can only be given in the present lived by the subject in question, in their individuality, along with co-subjects who share the dimension of intersubjectivity, experiencing the historical *a priori of Lebenswelt*; language - expressing intentionality, evidence, senses, meanings; reaching our historical horizon through tradition. From Gadamer we took the concept of hermeneutics. By understanding ways of interpreting texts, the horizon of comprehension of our historical-cultural ground was opened; the retroactive way of investigating scientific knowledge, already pointed out by Husserl, through possibilities of traversing paths opened by the question posed, which unfolds in questions and answers. We also worked with Heidegger's ideas of calculative thinking and meditative thinking.

Let us begin in the present. Studying algebra, we came across authors who approach it from both historical and philosophical points of view, as well as that of mathematical production.

Specifically, we examined Corry (1996), Van Der Waerden (1943), aiming to understand the construction and production of knowledge of algebraic structures. They point to important works by mathematicians who have contributed to the construction of algebra as it stands today, for example the works by Emmy Noether, Ernest Steinitz, Richard Dedekind, and Ernest Galois.

We proceeded to address a logically sustained question about algebra to the texts studied, while being aware of the logically sustained answers. we made such movement seeking to understand, describe and analyze those texts.

We assumed the Gadamerian hermeneutic posture and Husserl's view of history to elaborate a description of such movement of construction and production in terms of the historical universal *a priori* of tradition. The description and analysis worked on gave rise to a text we called the *solo text*[15] of our analysis as we continued to interrogate the studied texts about *the movement of thinking presented in the production of knowledge of algebraic structures*.

We carried out a retrospective analysis of the works of the above-mentioned authors and other important scholars, in search of what they have in common, that is, an invariant. We understand that complex numbers[16] constitute a *circumstantial driver of the notion of algebraic structure* as explained in Kluth (2005), through reflexive articulations, the phenomenological/hermeneutic analysis enabled us to understand the following *open categories*[17]: the modes of givenness of

[15] The solo-text was created based on studies conducted regarding the production of such authors when we examined their way of understanding and exposing algebra. We started from the present, that is, from chronologically situated authors and from the acceptability of their theories at present, closer to the period are living, considering mathematics, and looking at it from the standpoint of its body of knowledge. We approached the text with the following interrogation: how does the movement of construction/production of algebraic structures take place? We seek to understand the way the author tackled it in the text itself, advancing towards the moment through which the work of that author pointed to that of their predecessor. In other words, the mathematician whose theory extended to the point of accounting for the problems raised by them and their time in relation to what was being investigated. This procedure allowed us to understand the qualitive leap from one theory to another, that is, a change in the view of algebra to a more universal one.

[16] Other work regarding algebra consider it to be geometry. According to Wussing (1969) the genesis of the abstract concept of groups can also be investigated in the field of geometry.

[17] **Etymologically "Category"** comes from the Greek (*katēgoria*, predication, attribute), and Latin (*categoria*) and refers to the general concepts that demonstrate the various relations that can be established between ideas or facts. These general concepts become denominations that specify class, sets, etc., that characterize the objects that they may encompass. For instance, the group of individuals or entities that may be included in or referred to by a generic concept or conception; a class. Aristotle referred to "the general predicates attributed to the being, which correspond to distinct classes of being, as "categories of being". The phenomenological approach which we use, designates "open categories" as the articulation of invariants which converge to the same idea, however does not encompass predications of the phenomenon studied, (indeed it could not, in view of the meaning of phenomenon in phenomenology), but bring about possible openings for more understanding.

algebraic structures; the structures of the presence of both - algebra and the presence of human beings; and a way of being a mathematical human being. To illustrate this, we will present the open category of the *modes of givenness of algebraic structures,* exposing modes of analysis and articulation. From Kluth's research (2005), we will bring excerpts that explain important aspects of the intended goal.

IDEAS SUSTAINING THE ARTICULATION CONDUCTED

In the intertwining of our understandings of the ideas of Gadamer, Heidegger, and Husserl, we outline the investigative procedures that underpin our research. We will highlight some aspects of texts by these important authors to construct such procedures.

About Gadamerian hermeneutics. We understand that the hermeneutic attitude, as elaborated by Gadamer, requires a conscious attitude of the reader, so they can perceive and understand their opinions going towards, hearing the message of the text about the *thing*. "The task of hermeneutics becomes a question based on the thing itself and is always already determined by it" (GADAMER, 1997, p. 405). In Gadamerian hermeneutics, the historicity of the thing is characterized as the consciousness of effectual history, whose correlate in the experience of the other is to experience as the other, the alien, without disregarding the thing and what it has to say. One must be open to understanding the other as other than oneself, without annulling oneself in what they have to say and in the fulfilment of thinking. Gadamer focused on his own hermeneutic consciousness and enquires about its support, seeking to unravel its logic. His investigation led him to realize that all hermeneutic experience presupposes the logic that underlies the question. He sees the question as the art of maintaining the dialogue that advances dialectically towards unveiling what is said in the text. In such dialogue, the question and the answer are not ignored, but the aim is to expose their logical support, thus fulfilling understanding and interpretation. When understanding human work or text, it is the interpreter who understands and translates it into speech. This movement within oneself begins with the willingness to understand. However, this is not restricted to a solipsist movement; the interpreter is not positioned as a judge who, one-sidedly, authoritatively dispenses *truths* about what is said in the text. The hermeneut is open to what the text says but guided by the question that points to the latent answer in the work. They actualize the movement of thinking that occurs in the tension between uncertainty and perplexity that can occur when listening to the other and attentively assuming the experience. This is a thinking movement of understanding and interpretation that occurs in their subjectivity, being-with the text and being-with the other. Thinking is not an abstract action, devoid of materiality, but it is conducted in the opening of the horizon that is being unveiled.

When studying Discourse on Thinking (HEIDEGGER, 1962), we realized that the author speaks about two manners of thinking, calculative and meditative thinking. Both make sense, as we can see them in our daily lives and, more importantly, we realize that they are not exclusive. We understand that thinking both ways can guide us to broader understandings of the *how*, *why*, and *what for*.

For Heidegger, calculative thinking occurs when we are under any given conditions which we take into consideration to use for a specific purpose, whether for planning, researching, or organizing. Calculative thinking is not necessarily founded on numbers. Nonetheless, it is characterized much more by the human attitude towards data than by the very nature of data. It never stops. It goes from one program to the next, and so on. Conversely, meditative thinking is reflective and seeks understanding. Each person follows the path of meditative thinking according to their way of being and their horizons of comprehension. This is because, as Heidegger states, man is a thinking being, a meditative being.

The construction of meditative thinking was revealed to the authors of the present work, as a path we should follow to understand the construction of algebraic structures; in this sense, bringing out not only algebraic procedures and their justification, but emphasizing the elements that constituted it. From this perspective, the horizon of the meaning of thinking highlighted in the movement of production of algebraic structures changes. According to Kluth (2005), it transcends the domain of technical-scientific mathematical thinking deep-sited in Western civilization's in way of producing science and seeks the core of the intertwined notions present in what is called *algebra*. What does it say? What does it mean?

To understand the *historicity* and the *present* within the movement of constitution and production of algebra, we also worked with the ideas of Edmund Husserl. As a phenomenologist, Husserl does not work with the concept of factual history, which in a positivist way assumes the primacy of historical fact, naturalized and seen as objectively given. Historical fact is listed following events according to (chronological) dates and places. Husserl thought about what history is and stated that *from the beginning it is nothing more than the living movement of original meaning formation, and sedimentation of meaning with each other and within each other (Husserl, 1997, p. 457).* In his text, *Der Ursprung der Geometrie*, Husserl (1997) takes the term *origin* (Ursprung) as the original act, which is that of evidence, of seeing clearly what we intentionally want to understand. In many of his writings he refers to this act as taking place in the *now*, which is an instant and, as such, short-lived. Moreover, the evidence occurs in the scope of the subject's subjectivity.

However, we know that geometry[18], seen as a science of the Western world, is not maintained in this subjective scope, but extrapolates it and positions itself historically and culturally, lasting throughout generations. Key points to understand the leap taken in the movement of production of this science, from subjectivity to historical-cultural reality are: the experiences that show the way through which the apprehensions of time occur (Zeitauffassungen); the repetition of successful acts, which establishes the genesis of the identity of the structure throughout the repetition chain, intervening in this act the operation of idealization; the extrapolation to historical and cultural contexts in which we are faced with materialities that enable geometry to be presented in the Life-world; and the constitution of intersubjectivity through the acts of intropathy and language. Intropathy is knowledge of the other that occurs directly in the experiences through which the other is given (brought, exposed) to the self in their corporeality. It is a constituent perception of intersubjectivity. Therefore, it is not a theoretical concept or a predicatively constructed statement. Intropaphy allows us to transcend the strictly egological sphere and become aware or reach otherness. Language expresses the intertwining between what is intended, what is spoken out loud, what is understood in evidence, It is composed of sounds, senses, signs, and meanings[19] that enable its materialization in *Lebenswelt*. Writing strengthens and imposes a characteristic of "logical" activity, "specifically linked to language, as well as the ideal cognitive configuration that is specifically generated there" (Husserl, E., Anexo III, 2008, p. 380). The science that is there, in addition to geometry and mathematical disciplines, come to us by tradition conveyed by various modalities of language.

Mathematical disciplines are human achievements. Their origin has been established as a first acquisition originated from creative subjective activity made possible by an original evidence grounded in the Life-world. However, they are subject to a continually synthesizing dynamism in which all previous acquisitions remain valid, forming a totality, such that, in every total presence, as historically constituted meaning, given in the syntheses, rests the total premise for the acquisition of a new level. This way, the meaning of the previous acquisitions is preserved, allowing us, from the present, inquiringly, in a retrospective movement, to understand the original evidence, the movement of their production, their historical way of being and what they tell us today, in this present we are living.

[18] Geometry is taken by the author as solely an example of the way of being of all theoretical sciences, or as some call them, "pure" sciences.
[19]Kluth (2005, p.48), referring to Derrida (1994, p. 27), states that the word meaning, *Bedeutung* in German, and the word sense, *Sinn* in German, are used by Husserl with different functions: one refers to language and the other to intuited or perceived objects. *Bedeutung* is reserved for the ideal meaning content of verbal expression, of spoken speech, while sense (*Sinn*) covers the entire noema sphere even in its non-expressive layer.

ANALYZING AND ARTICULATING MEANINGS

Hermeneutic analysis guided us to articulate the *structural invariants*[20] as parts of the solo-text, which are grouped under *the modes of givenness of algebraic structures*. This pointed to the *a priori* structure of algebraic structures in the flow of the *historical universal a priori* that reveals its historicity. This historicity is conveyed in the fabric of its temporality as *living present* that does not address things, but rather their modes of givenness that constitute a fabric of retentions and protensions.

Understanding the Gadamerian ideas about the dialectical and dialogical movement of asking and answering, the Heideggerian ideas about meditative thinking, the phenomenological ideas on temporality, history and language that permeate the fabric of the ideality of *algebraic structures,* we began, as previously stated, to articulate the excerpts of the solo-text. This solo-text answers the question: "*what is the way of being of algebraic structures?*". This question examines the ways of givenness of algebraic structures in the perspective of the *nows,* highlighting the *structural invariants* and the *living present* as it focuses on the *referral system*.

As a reference to understanding the analysis carried out, here, we will register a portion of the solo-text that answers the question about the above-mentioned modes of givenness.

Dedekind starts from the following observation: "given a rational number a, let us consider A_1 the class of all rational numbers smaller than a and A_2 the class of all rational numbers greater than a. Take a such that it belongs to A_1 or A_2, so either a is the largest number of A_1, or a is the smallest number of A_2." We can then state that:

1 -A_1 and A_2 are disjoint.
2 - Every rational number belongs to A_1 or A_2.
3 - Every number of A_1 is smaller than any number of A_2.

What is the way of being of *algebraic structures?*

Two classes that satisfy the three properties mentioned are called *cuts*. <u>Thus, Dedekind introduced his most important conceptual innovation, that is, properties are used as instruments to define a given numerical circumstance</u> [31 P1]. Dedekind shows that any rational number defines a *cut,* but not every *cut* is defined by a rational number, which means a certain discontinuity in rational numbers and that a continuous system is the collection of all *cuts*, as the straight line. He defines the real numbers as the collection of all rational cuts and demonstrates all the properties of this new system, exclusively by using the inclusion relation. It discusses the order properties and shows that the real number system is a fully ordered system and that it forms a continuum. Finally, he defines

[20] *Structural invariants*, in phenomenology, refer to the essential characteristics of the phenomenon analyzed. They show, at each moment of the investigation, the structure of what is researched, that is, the 'structure' of the structures of algebra that is presented in the texts, that expresses it and is hermeneutically understood and interpreted.

> all real number operations and proves their properties. This study of real numbers underlies operations such as $\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$. (KLUTH, 2005, p. 98)

After highlighting all the responses, indicative of the *structural invariants* present in the solo-text, they were grouped and interpreted from the perspective of the *nows* and the *living present*. Some insights from the analysis are presented below.

In the works of Galois and Dedekind, algebraic structures are presented as algebraic *structural notions* that fulfil an instrumental purpose. They are configured as resources for studying mathematical objects, articulating genuine and characteristic principles of the field of arithmetic. Thus, algebraic structures play the role of boundary delimiters in the numerical field by defining properties that do or do not occur in the field considered, as in the case of hypercomplexes, where $a.b = 0$ does not mean that a or b are null.

On the other hand, algebraic structures are revealed in their integrative characteristic mediated by operational principles that unite the regions determined by the boundaries. The relational integrative characteristic of algebraic structures, whether by principles, properties, formation law or the inclusion relationship present in the definitions of different types of structural notions, reveals a change in the field of algebra, interpreted by some non-phenomenological scholars, such as Wussing (1989), as a change in the way mathematics is done as a science.

> However, this change in the field of algebra, when analyzed under the phenomenological prism that describes the modes of givenness of the temporal object, denotes another way of presenting numbers. Numbers are given otherwise than that of counting, calculating, measuring. Numbers are mediated by their structurers, by what constitutes them as numbers or numbers of a given numerical class. (KLUTH, 2005, p. 144-145).

Thus, the notions express the *a priori* structure, the original seed of phenomenalization of algebraic structures within the numerical field, reaching new modes of givenness, becoming themselves the object of study in the body of algebraic knowledge. This characteristic is identified in the work of Steinitz (1950) as a theorization moment, when the properties and laws, generators of notions, are taken as axioms. Its purpose was to gather all types of numerical bodies, reinforcing the integrative and relational mode of being of algebraic structures. In the evaluation of scholars, this culminated in the axiomatization of classical algebra, as it indicates a mathematical variety as the characteristic of being of algebraic structures.

Still on the trail of protensions, algebraic structures become the subject of algebra. This leap of objectification can be seen in the work of Noether, which unifies species of structures around principles, such as the uniqueness of *ring factorization,* which gives rise to the Multiplicative Ideal Theory whose conducing thread is factorization.

> In Noether's work, a real boundary is manifested between numerical formalization and the formalization of algebra as the unification of all algebra, not only through the equality of certain properties of certain sets, but around possible articulations which correspond to various forms of factoring. (KLUTH, 2005, p. 15).

It is in this development stage of *algebraic structures* that reside their seeds, as a *structure* that can be found in sets of elements of a different nature. Thus, the *structure "group"* no longer refers to the numeric set, but also for instance to the set of transformations, albeit, each such sets retains its own operational mode. In addition, it is revealed that structural domains maintain a relationship of inclusion, which defines a hierarchy expressed axiomatically around operational laws.

Following the path of its becoming (*devenir*), the structures, with this conception dealt with by algebra, become objects of mathematics and are seen with the purpose of providing the basis for all mathematical knowledge, preserving their relational integrative character. Through the structural approach, mathematics gains spaces for application that transfer the idea of *structure* to other regions of inquiry.

In the wake of actualizations of the formation movement of constitution and production of the ideality of algebraic structures, temporality can be seen in terms of *nows*, *past-nows* and *future-nows*. This movement shows the development of mathematical achievements by weaving a unity of the multiplicity of temporal phases. These turn out to be, as seen by the eye, the algebraic structures from the perspective of the *living present*.

To speak of algebraic structures as subject to a process of maturation is to assume them in their way of being as *a being that is, and that constitutes themselves in being when giving themselves*. The formation of this ideality takes place in the stream of intentionalities, which merge into intersubjectivity by being with others in a reference system that carries not only the content of a now, but also the synthesis of all *nows* that include proofs, validations and assents; which describe its original way of being and the way the modulation to which it belongs takes place. In this case, the modulation that articulated, articulates, and will articulate the maturation of algebraic structures is Western mathematics.
Algebraic structures, when analyzed from a Husserlian phenomenological perspective, retain their meaning in the most intimate of human actions. Structures are revealing of vital movements around invariants intertwined with a theoretical practice that occurs within mathematical modulation.

In the statements presented above, under the phenomenological approach, *algebraic structure*s stem from the fact that phenomenologically investigating the first presentation of the formation of ideality temporalized in space, in the intentional relation men-world, not only aims to understand the instance responsible for making objects *appear,* which for phenomenology is *subjectivity* and all intentional acts that constitute this appearance, but also contemplate the so-called *horizons of intentionality*. That is, when one becomes aware of the object, one also becomes aware of the world.

The intentionality of this act and the horizons of intentionality added to the understanding that "the very notion of phenomenon, of object in the *how* of its way of being given, already designates a *categorial objectivity*¨ (Husserl (1987), pp. 38 and 142 apud Moura (2000) p. 230). This means, for example, that the perception of a red object, not only alludes to the presence of that object with that color, but also to the redness that defines the category of the color red. Such redness may be present in any other object, different from that which was seen. This opens possibilities for numerous other relationships that redness entails. Thus, this encompasses the external horizon of perception analysis.

The *horizons of intentionality* reveal that objects are seen from various perspectives, as well as different ways of giving. The same happens with mathematical objects taken in the sedimentation of layers of objectivation characterized by Husserl as:

> (1) that of the mundane objects, situated in objective time; (2) of the "internal" objects, as intentional acts and sensations, that unfold in an immanent temporality to consciousness; (3) ultimately, the sphere of absolute consciousness which constitutes time itself, that to which an object ¨appears¨ as a temporal object. (Husserl (1966 b), p. 73 apud Moura p. 232)

This most intimate nucleator process, that is, more internal to the relation between humans and the world, from which human relations derive. Including those actions which put us before *algebraic structures*, or in the presence of numerical properties while "dealing" with numbers. The awareness of the properties, that is, the realization of the properties, cannot be taken independently from the whole. For instance, this can be viewed as a numeric set, even though the properties present differently from the whole. From a phenomenological perspective, the perception of an object and its properties is understood as an intertwining of senses, evidencing that it is articulated in terms of retentions and protensions, from visible to invisible. Thus, the changes that occurred throughout the historicity of algebra, as previously stated, are not a change in the way mathematics is produced as a science. However, it is about a new view of what is already known and instituted in the body of mathematical knowledge, intuiting, through clear evidence, possibilities of change.

When our aim is teaching and learning mathematics, and focus on the transition from arithmetic to algebra, understanding the change in view as a result of meditative phenomenological thinking, leads us to envision moments of mathematical experiences that may circumvent or minimize certain difficulties with which both teachers and students are already familiar, such as those that have to do with the idea of substitution that involves the meaning of letters and variables. Booth (1997) points out that in arithmetic numeric symbols always represent the same value. As a result of such assertion, many students understand that there is a direct *letters* *numbers* correspondence, where, similarly to numbers, letters would also represent quantities. Thus, students could assume that different letters must necessarily have different values. However, the transposition of numbers, seen as quantities, to letters, encompasses a change in structure. One is operating with a different view of reality: instead of looking at and working with specific numbers and the respective quantity, for instance 3, now, one can work with the universality of "any given value", *a* entering the realm of *numerical properties which* stem from dealing with numerical values.

An example is the proposal by Bernhard (1991), that, as we understand shows, under a phenomenological view, the notion of algebraic structures. We intend to open possibilities for the student to realize the regularities in the passage from arithmetic thinking to algebraic thinking. The movement of teaching and learning activities is furthered as we advance from calculative thinking to meditative thinking. That is, for instance, we take the distributive property and work towards clarifying the way it is operated, then, seek to understand what it says, which ideas are being worked on.

Explaining the example:

The known distributive property of natural numbers initiated by:

$7 . 23 = 7.(20 + 3) = 7.20 + 7.3 = 140 + 21 = 161$ and extended to $32 . 43 = (30 + 2). (40 + 3) = 30.40 + 30. 3 + 2 .40 + 2 . 6 = 1200 + 90 +80 + 6 = 1378$, whose results can be confirmed, using algorithms of addition and multiplication operations with which the students are already familiar.

Gradually, numerical regularities can be introduced, by requiring possible organization of what is seen, that can be entered into a table, such as:

$12. 13 = (10 + 2) . (10 + 3) = 100 + 30 + 20 + 6 = 156$

$22 . 23 = (20 + 2) . (20 + 3) = 400 + 60 + 40 + 6 = 506$

$32 . 33 = (30 + 2) . (30 + 3) = 900 + 90 + 60 + 6 = 1056$

In the movement of hermeneutic comprehension that can be conducted between the student and the teacher, we ask what can be seen in this representation, specially what is pointed out. In the expressions listed above, there are numerically expressed repetitions and both operational and non-operational regularities that can be expressed in other ways, through mathematical language. How? In the present text, we are dealing with possible situations. Among which can be highlighted, for instance: $12 . 13 = (10 + 2) . (10 + 3) = 10^2 + 3 . 10 + 2 . 10 + 2.3 = 156$. The same thing can be seen in $(20 + 2) . (20 + 3) = 400 + 60 + 40 + 6 = 20^2 + 3. 20 + 2. 20 + 2.3 = 506$ and so forth if we take 32.33; 42. 43, etc.

This is an activity in which numerical repetitions can be highlighted. What do they mean? The thinking movement can conduce to the understanding of regularities related to the first numbers of number decompositions which must be multiplied. Possibilities for the understanding of regularities are thus opened.

How can this be universalized? Posing this question to students leads to theorizing thought that transforms first understandings into a more encompassing whole, organized by specific operations. This break-through that modifies the view can be described as follows: any two given numbers, expressed in tens and ones can be decomposed in tens added to their ones.

For example: $12 = 10 + 2$; $13 = 10 +3$; $22 = 20 + 2$ and so forth.

A specific regularity of these numbers is evident. They can be decomposed in tens (10; 20; 30 ; ...) + ones (1, 2, 3, ...) the numbers 1, 2, 3 are present in all the expressions. They are fixed. However, the value of tens may vary. Therefore, the expression that gathers all the tens + the ones can be written as $(a + 2)$ and $(a + 3)$ and its multiplications as $(a + 2) \boldsymbol{.} (a + 3)$.

The same movement can be conducted with other expressions. For instance: $20^2 + 3. 20\ 2. 20 + 2.3$. In that expression the first ten is squared and added to its multiplication by the fixed 3 and 2, as well as the multiplication 2.3. this happens in all cases shown on the table.

There is a regularity which is the focus of our inquiring gaze while seeking the modes of categorial expression, when viewed within the mathematical dimension, that is presented to us in the present time, through books and texts, for example.

Thus, we have: $(a + 2) . (a + 3) = a^2 + 3. a + 2. a + 2.3$.

What does these activities bring? A look at numbers from a different perspective. Yes, there they are. And what else? In this activity, the presence of numbers is revealed and also brings the presence of their properties and principles as elements that define a certain category. Such category, in turn, requires the elaboration of an expression and, with it, the basis for a new linguistic resource.

A COMPREHENSIVE SYNTHESIS OF THE ARTICLE

We have presented our understanding of aspects of Gadamerian hermeneutics, of Heidegger's thinking, and history in Husserl. The fabric of such understandings enabled us to outline procedures for a phenomenological-hermeneutic analysis through meditative thinking, nourished by the questions addressed to the algebra of the present, which also comes to us through written texts that indicate answers to those questions.

We look at the algebra of the historical present of the Life-world of mathematics seen as a Western science. The present is there, mentioned in texts by significant scholars who built abstract algebra, exposed in history and mathematics books. The hermeneutic-phenomenological work took place in the wake of the actualizations of the formation of ideality of algebraic structures exposed in the texts of such authors, made present in their nows, past-nows and future-nows. We highlight the maturation of mathematical achievements by weaving a unity of the multiplicity of temporal phases. These turn out to be what is seen by the eye, algebraic structures from the perspective of the *living present*.

In this investigation we could observe the thinking that reveals itself in the construction of the knowledge of algebraic structures, which extends far beyond calculative and technical thinking. It shows its autochthony from its genesis, whose revealed source is detailed by complex numbers in their ways of being and continuing to be, in their modes of expression and of organization that contemplate both the scientific and cognitive processes constituting layers of objectification of the ideality of algebraic structures. This understanding leads us to question the reason mathematics education has been proposing teaching mathematics based solely on the understanding of mathematical results via calculative thinking.

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